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INTERPOLATION OF GROUNDWATER LEVELS USING KRIGING IN SAGAR DISTRICT (M.P.)

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ABSTRACT

Groundwater is one of the major source of water all over the world. Management of this resource is very important to meet the increasing demand for domestic, agricultural and industrial use. Various management measures needs to know the spatial and temporal behaviour of groundwater.

Also most of the groundwater models require the input to be available on a grid pattern. But in the field, these parameters are generally measured at random points. So, interpolation of parameters at the grid nodes is a prerequisite to the use of this data in groundwater modelling.

Interpolation of ground water levels is of significant importance in agricultural and hydrologic contexts. Water recharged into ground water is of prime importance in agriculture as it can be conveniently tapped during the dry season. However, if the water table rises to the root zone depth, so as to adversely affect the yield of the crop, the land is said to be waterlogged.

A variety of interpolation techniques are in use. The most widely used methods in this context are polygonal methods, triangulation methods, isoline methods and kriging techniques. Polygonal methods are based on the principle of Voroni neighbourhood, wherein, the magnitude of the entity at the point under consideration is the same as that at the geometrically nearest measured point. The main drawback in such a scheme is the spatial discontinuity in the concept. Triangulation methods overcome this drawback by considering spatial continuity on the plane generated by the magnitudes of the entity and these observation points. In the isoline method, isolines for the entity are drawn using linear interpolation techniques. Various studies (Vieira et al, 1981; Yost et al, 1982; Knighton and James, 1985; Dahiya et al, 1986; and Laslett et al, 1987) have shown that kriging performs better than the other above mentioned methods.

In this report, an application of kriging technique is shown to interpolate the groundwater levels as measured in Sagar District of Madhya Pradesh.

1.0 INTRODUCTION

All over the world, groundwater is one of the major sources of water. However, the dynamics of groundwater is affected by a number of natural and anthropogenic factors. Prominent amongst the latter are manmade storage projects, withdrawal of groundwater, urbanisation projects, mining projects etc.

Appropriate management measures can reduce the undue building up or depletion of groundwater. Some of the management measures may be in the form of restraints on the user, while some may be in the form of engineering measures. To be sound and acceptable, both these methods need reliable quantitative information on groundwater behaviour. That is to say, estimation of the likely water levels, at various locations in space and time, the annual recharge/depletion potential, the flow direction etc are the basic inputs on which management measures must be built on. There exists a need to study the spatial and temporal behaviour of groundwater.

With the advent of high speed computers, which can handle large volume of data easily, more and more distributed models are becoming available. Results of these modelling attempts are only as good as the input information. According to Mulla (1988), a major ongoing challenge to watershed modelling is to treat the spatial variation in soil factors more quantitatively, and reduce the extent of spatial averaging that is implicit in the present models. According to Rehfeldt et al (1992), a means of quantifying the spatial variability, for example of aquifer hydraulic conductivity which controls the movement and dispersion of groundwater solutes, at a reasonable expense is essential for the application of solute transport models to practical problems. Selection of the correct resolution in these modelling at the watershed scale is also essential in reducing scale related errors (Farajalla and Vieux, 1995).

Presently, observed groundwater levels serve as one of the main input data in studies related to groundwater simulation, water balance, groundwater recharge potential, groundwater estimation and in the design of drainage structures. Of the various measurement practices, measurement of groundwater level is the simplest and the most economical. However, these measurements are generally carried out randomly in the field. Most of the groundwater models requires these measurement at a prespecified grid. So, there is always a need to interpolate the measured parameters at the grid points.

A variety of interpolation techniques are in use. The simplest methods, arithematic mean method, nearest neighbour method, distance weighted method, and polynomial interpolation uses distance between data points to relate the change in the magnitude of the entity. However, geophysical attributes exhibit some spatial structuring (Delfiner and Delhomme, 1975; Huijbregts, 1975). This spatial structure, if deciphered and incorporated in model building, can improve the quality of interpolation (Sabourin, 1983). Geostatistics, a set of statistical techniques , is such a technique which takes into consideration the spatial structure and so scores over the other methods.

Geostatistical technique was used to study the spatial variability of groundwater depth data by Dahiya et al (1986). It was concluded that these contour maps could serve as a better background for making appropriate decisions in the management of groundwater in the area.

Spatial variability of infiltration rate was studied by Vieira et.al, (1981) using geostatistical technique. In this study it was concluded that contour map obtained using kriging technique showed a smooth geographical pattern of infiltration whereas contour map by polynomial interpolation, showed abrupt changes.

Yost et al (1982) have found that the kriged map of soil P sorption corresponds more closely with known soil behaviour than did the spline map.

Knighton and James (1985) applied geostatistics to study the spatial variability of soil test phosphorus (STP) and concluded that Kriging performed better than fifth order bivariate polynomial.

Laslett et.al. (1987) compared the performance of several two-dimensional spatial prediction methods for predicting soil pH. The methods compared were global means and medians, moving averages, inverse squared distance, Akima's interpolation, natural neighbour interpolation, quadratic trend surface, Laplacian Smoothing splines and ordinary Kriging. Laplacian smoothing splines and Kriging were found to generally perform best.

Di et.al. (1989) has concluded that geostatistical approach is more efficient (in terms of number of samples) than conventional statistical methods in designing sampling strategies.

Gallichand and Marcotte (1993) compared the accuracy of estimation for soil clay content using different spatial interpolation methods and found that geostatistical techniques performed better than other interpolation methods.

In this report, kriging, (Journal and Huijbrets, 1978; Devi and Kumar; 1994, 1995; Oleo, 1974), a type of geostatistical technique is applied to interpolate the groundwater levels as observed in the Sagar district of Madhya Pradesh, India.

2.0 METHODOLOGY

Geostatistical technique is used in this report to interpolate the groundwater level data. A brief theory of the technique is given in the following pages.

2.1 Geostatistics

According to Matheron (1963) "Geostatistics is the application of the formalism of random functions to the reconnaissance and estimation of natural phenomena". It can be described as a systematic approach for making inferences about quantities that vary in space. Geostatistics is based on the Theory of Regionalized Variables.

When a variable is distributed in space, it is said to be "regionalized" (Journel and Huijbregts, 1978). A Regionalized Variable (ReV) is defined by Matheron (1963) as the variable that spreads in space and exhibits certain spatial structure. Such variables show a complex behaviour. Their variations in space are erratic and often unpredictable from one point to another; however, these are not completely random as these exhibit some spatial correlation. So, a ReV is simply a function of space, but generally a very irregular function. All the parameters generally used in groundwater hydrology, such as transmissivity, hydraulic conductivity, piezometric heads, precipitation, vertical recharge etc. can be called regionalized variables.

A ReV possesses two contradictory characteristics (Journel and Huijbregts, 1978), a local random, erratic aspect and a general structural aspect. This twin aspects of randomness and structure have been taken into account in the theory of regionalized variable by considering it to be based on probabilistic theory of random functions. A random variable (RV) is a variable which takes a certain number of numerical values according to certain probability distribution (Journel and Huijbregts, 1978).

Let Z be a property of the aquifer, say the piezometric level; then, Z(x) is defined as a random function (RF) where x represents the coordinates in 1,2, or 3 dimensions. $z_1(x)$ is called a realization of Z(x). The function Z can have an infinite number of realizations i.e. $z_1(x)$, $z_2(x)$, $z_3(x)$......

Some assumptions are introduced to develop a working model that can be used for estimation purposes. These assumptions are

- The RF Z(x) exhibit some kind of stationarity.
- The RF Z(x) is ergodic.

In linear geostatistics, only the first two moments of the RF are used and so, it is sufficient to assume the second order stationaritity. These conditions implies that variance exists and is finite. In practice, this assumption is restrictive. So, the second order stationarity hypothesis can be relaxed and a weaker hypothesis, known as Intrinsic Hypothesis, can be considered.

This hypothesis assumes that the first order increments of the RV's are themselves second order stationary i.e. for any two RV's, Z(x) and Z(x+h), separated by a distance h, the increment Z(x+h)-Z(x) has zero expectation and a finite variance which is independent of the point x. The variance of the increment defines a new function known as Variogram i.e.

$$E\ \{Z(x+h)\ -\ Z(x)\}\ =\ 0 \tag{1}$$

and

$$Var\ \{Z(x+h)\ -\ Z(x)\}\ =\ E\ \{[Z(x+h)\ -\ Z(x)]^2\}\ =\ 2\gamma(h) \tag{2}$$

where,

$2\gamma(h)$ is defined as the Variogram.

$\gamma(h)$ is known as semivariogram.

A system which satisfies the stationarity of order two, also satisfy the intrinsic hypothesis, but the converse is not true.

The estimator $2\gamma(h)$ is the arithematic mean of the squared difference between two experimental measures, ($Z(x_i)$ and $Z(x_i+h)$), at any two points separated by the vector h. For a set of N sample values,

$$2\gamma^*(h) = \frac{1}{N(h)} \sum_{i=1}^{N(h)} \left[z(x_i) - z(x_i+h)\right]^2 \qquad (3)$$

Where,

N(h) is the number of experimental pairs separated by vector h in the data.

The semivariogram is calculated as

$$\gamma^*(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left[z(x_i) - z(x_i+h)\right]^2 \qquad (9)$$

A plot of $\gamma^*(h)$ versus the corresponding value of h, also called the semivariogram, is thus a function of the vector h, and may depend on both the magnitude and the direction of h. A sample plot of semivariogram is shown in Fig. 1.

The distance at which the variogram becomes constant is called the range, a. The value of the semivariogram at a distance equal to the range is called the sill. This, value

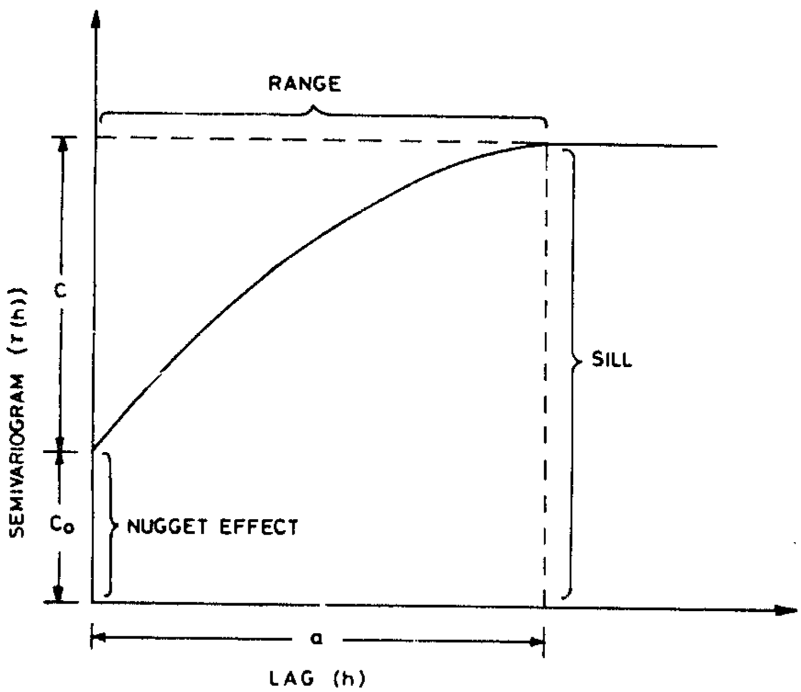


FIG. 1 PLOT OF SEMIVARIOGRAM

is simply the a prior variance of the RF. Semivariograms may also increase continuously without showing a definite range and sill. Such types either correspond to RF which are only intrinsic or indicates the presence of non-stationarity. The value of the semivariogram at extremely small separation distance is called the nugget effect.

The behaviour of the semivariogram near the origin and at infinity are two other important features of the semivariogram as these express the qualitative characteristics of regionalization (Matheron, 1963).

Behaviour near the origin characterizes the continuity of the ReV. The regularity of ReV is represented by the more or less regular behaviour of $\gamma(h)$ near the origin. Examples of the four classical types of behaviour are shown in Fig. 2. Fig. 2(a) has a parabolic shape near the origin and presents a ReV with high continuity such a the head in a deep observation well as a function of time. (Delhomme, 1978). Linear shape (Fig. 2(b)) represents a ReV which has an "in average" continuity. Discontinuity at the origin corresponds to a variable presenting not even an "in average" continuity (Matheron, 1963). Two distinct points at a very close distance will also show a difference. Fig. 2(c) shows this nugget effect. Fig. 2(d) shows a pure nugget effect and it is the limit case when the semivariogram appears solely as a discontinuity at the origin (Journel and Huijbregts, 1978). It corresponds to a total absence of auto correlation and hence pure random behaviour.

The semivariogram increases more slowly as lag distance 'h' tends to infinity. An experimental semivariogram, which increases at least as rapidly as $|h|^2$ for large 'h' does not hold the intrinsic hypothesis and indicates non-stationarity (Journel and Huijbregts, 1978).

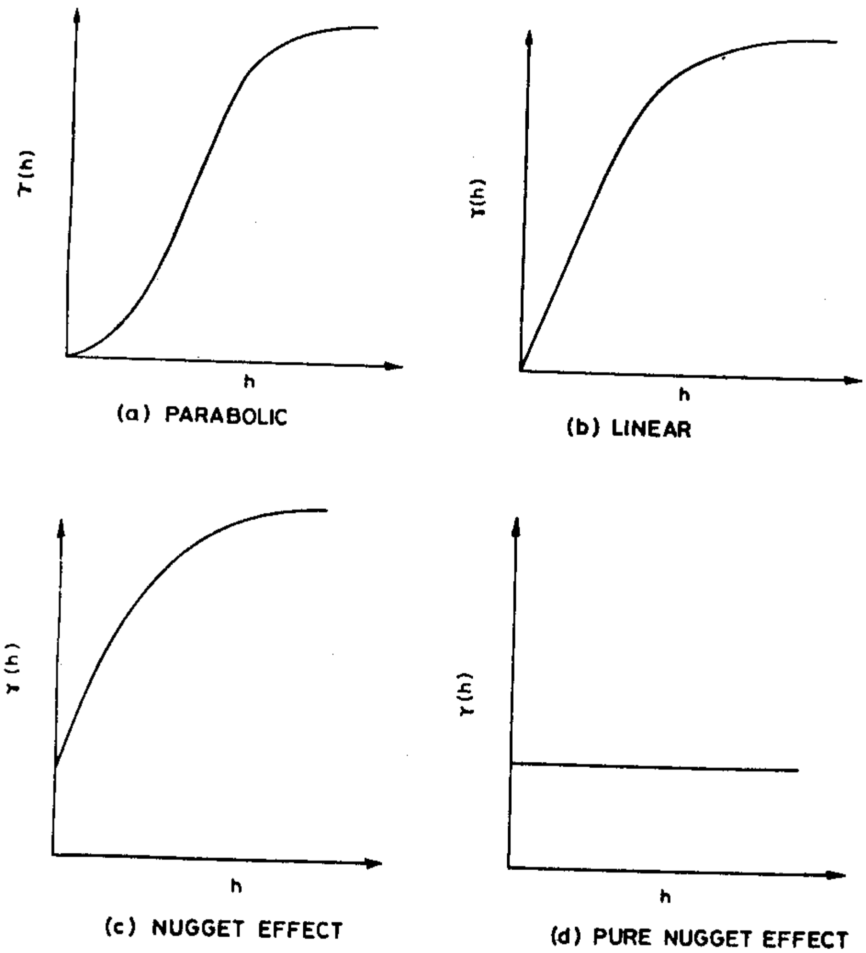


FIG. 2. BEHAVIOR OF THE SEMIVARIOGRAM NEAR THE ORIGIN

The semivariogram, given in Eq.3 is also termed the true semivariogram of the ReV. As only one realization of the RF is available, the true semivariogram can only be estimated and this estimate is known as the experimental semivariogram.

If the sampling is done on a regular grid, the $\gamma^*(h)$ may be estimated for values of h, known as lag distance or lag increment which are multiples of the grid spacing. This situation is rare in practice, particularily in the context of groundwater and the chance of finding pairs at exactly same specified distance h is very small. To overcome this, a tolerance, δh is placed on the distance. Every pair of observations that are separated by a lag $h \pm \delta h/2$ are then used to estimate $\gamma^*(h)$.

The above procedure, is used for calculating the isotropic experimental semivariogram, also known as omnidirectional semivariogram. In this case, it is assumed that the variation is the same in every direction. To find the anisotropies, the semivariograms are calculated in different directions. To do this, a tolerance, $\delta\theta$, is placed on the directional angle.

The experimental semivariogram has discrete values and irregular shape due to the limited sampling. A mathematical function used to approximately represent this semivariogram is known as the theoretical semivariogram. The process of fitting a theoretical model to experimental semivariogram is called structural analysis. This process is the first and most important step in the geostatistics as it affects the final results.

For any function to be a valid function for a semivariogram, it should meet the positive definite condition. It is safe to use only those functions which are tested and are used in literature. These functions are varied enough to enable a satisfactory fit to all sample variograms likely to be encountered in practice. Some of these models are discussed below.

Spherical model :- (Fig. 3(a))

$$\gamma(h) = \begin{cases} C_0[1-\delta(h)] + C\left[\frac{3}{2}\frac{h}{a} - \frac{1}{2}\frac{h^3}{a^3}\right] & h \leq a \\ C_0 + C & h > a \end{cases} \quad (4)$$

Exponential model :- (Fig. 3(b))

$$\gamma(h) = C_0[1 - \delta(h)] + C\left[1 - \exp(-\frac{h}{a})\right] \quad (5)$$

Gaussian model :- (Fig. 3(c))

$$\gamma(h) = C_0[1 - \delta(h)] + C\left[1 - \exp(-\frac{h^2}{a^2})\right] \quad (6)$$

Linear model :- (Fig. 3(d))

$$\gamma(h) = C_0[1 - \delta(h)] + bh \quad (7)$$

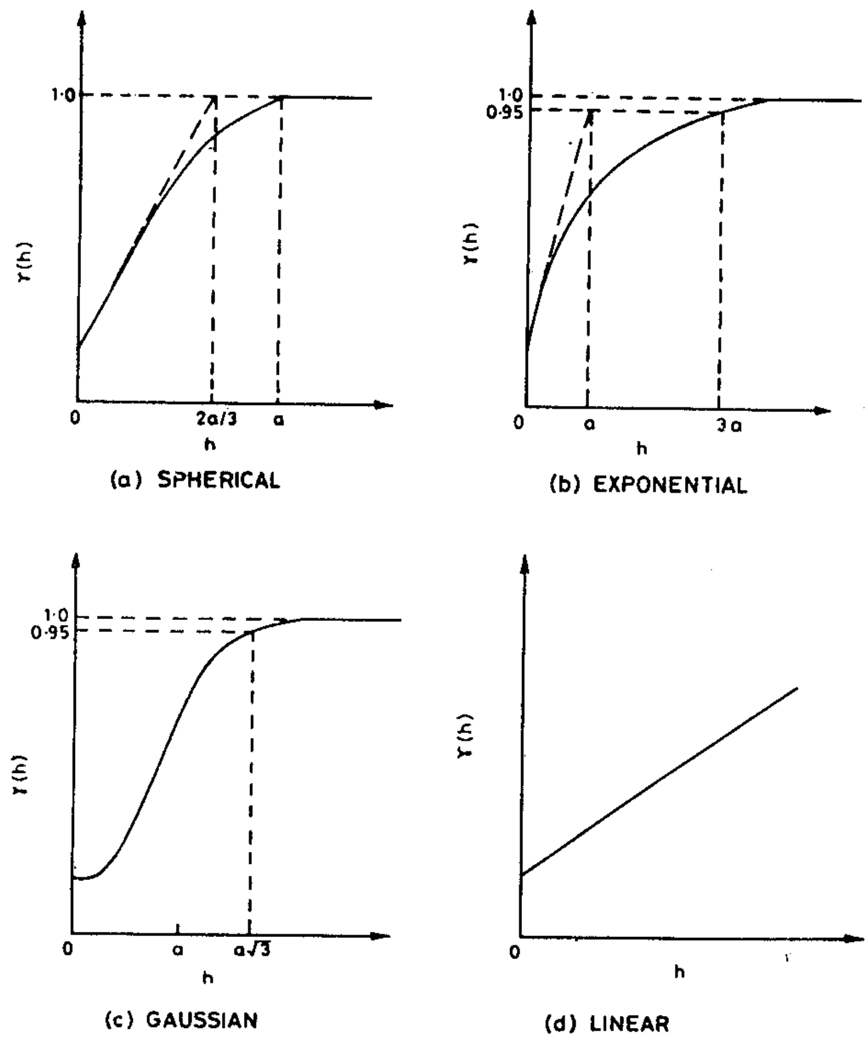


FIG. 3 THEORETICAL MODELS OF SEMIVARIOGRAM

Where,

$$\delta(h) \text{ is the Kronecker delta} = \begin{cases} 1 & h = 0 \\ 0 & h \neq 0 \end{cases}$$

C_0 is the Nugget effect

C_0+C is the Sill

and a is the Range and b is the slope at the origin.

2.2 Kriging

According to David (1977), kriging, a technique developed by D.G. Krige for application in South African gold mines, covers both the best linear unbiased estimator (BLUE) at a point and the best linear weighted moving average of a block. It is a technique of making optimal, unbiased estimates of regionalized variables at unsampled locations using the structural properties of the semivariogram and the initial set of data values. Kriging can be applied to estimate the value of a variable at a particular point, (Punctual Kriging) or to estimate the average value of a block (Block Kriging). In punctual kriging, by changing the position of the point, it is possible to estimate the whole area of interest.

Consider a situation in which a property is measured at a number of points, x_i , within a region to give values of $z(x_i)$, $i=1,2,3,\ldots,N$. (x_i is the coordinate of the observation point in 1, 2 or 3-dimensional space). From these observations, the value of the property at any place x_0 can be estimated. The place might be a 'point', i.e. an area of the same size and shape as those on which measurements were made, or a larger area or block. Such situations commonly arise in hydrology. For example, in the estimation of rainfall, temperature, sunshine; in the estimation of hydrological parameters such as transmissivity, piezometric head, solute concentration in a plume etc.

Linear geostatistics estimates the kriged value of z at x as the weighted sum of the measured values i.e. for point estimation.

$$z^*(x_0) = \sum_{i=1}^{N} \lambda_i z(x_i) \qquad i=1,2,3,...,N \tag{8}$$

where,

$z^*(x_0)$ = estimated value at x_0

λ_i = weights chosen so as to satisfy suitable statistical conditions

$z(x_i)$ = observed values at points x_i

Apart from providing the estimate of a property, geostatistics also provide the estimation variance which measures the accuracy of the estimate.

By taking $z(x_i)$ as a realization of the random function $Z(x_i)$ and so $z(x_0)$ as the realization of $Z(x_0)$, the Eq.8 can be written as

$$Z^*(x_0) = \sum_{i=1}^{N} \lambda_i Z(x_i) \qquad i=1,2,3,...,N \tag{9}$$

In kriging, the weights λ_i are calculated so that $Z^*(x_0)$ is unbiased and optimal.

$$E\{Z^*(x_0) - Z(x_0)\} = 0 \tag{10}$$

The condition of optimality means that the variance of the estimation error should be minimum i.e.

$$Var\{Z^*(x_0) - Z(x_0)\} = minimum \tag{11}$$

substituting Eq.9 into Eq.10 leads to

$$\sum_{i=1}^{N} \lambda_i = 1 \tag{12}$$

for stationarity of order-2. The estimation will be unbiased if the Eq.12 will hold.

Substitution for $Z^*(x_0)$ in the minimum variance condition and rearrangement of resulting terms in terms of $\gamma(h)$ and $\gamma(0)$ yield :

$$E\{[Z^*(x_0) - Z(x_0)]^2\} = -\sum_{i=1}^{N}\sum_{j=1}^{N} \lambda_i \lambda_j \gamma(x_i,x_j) + 2\sum_{i=1}^{N} \lambda_i \gamma(x_i,x_j) \tag{13}$$

The Eq.13 is a quadratic function of the weights λ_i. The minimization of the above function, subject to the linear constraint of Eq. 12 is found using the Lagrange multiplier, μ and taking the partial derivatives for all λ_i, i.e.

$$\frac{\delta}{\delta \lambda_i}\left[E\{[Z^*(x_0) - Z(x_0)]^2\}\right] - 2\mu\left[\sum_{i=1}^{N} \lambda_i - 1\right] = 0 \tag{14}$$

Where,

μ = Lagrangian multiplier

On simplification of Eq. 14

$$-2\sum_{j=1}^{N} \lambda_j \gamma(x_i,x_j) + 2\gamma(x_i,x_0) - 2\mu = 0 \tag{15}$$

Rearranging and combining with Eq. 12 results in the kriging system equations.

$$\begin{cases} \sum_{j=1}^{N} \lambda_j \gamma(x_i,x_j) + \mu = \gamma(x_i,x_0) & i=1,2,3,...,N \\ \sum_{j=1}^{N} \lambda_j = 1 \end{cases} \tag{16}$$

Substitution of Eq. 16 into Eq.13 yields the estimation variance, $\sigma_k^2(x_0)$ at x_0, as:

$$\sigma_k^2(x_0) = \sum_{i=1}^{N} \lambda_i \gamma(x_i,x_0) + \mu \tag{17}$$

Solution of the above set provides the values of λ_i which can be used for estimation.

The kriging technique developed for the estimation of non-stationary ReV is called universal kriging. There are many other variations in kriging such as cokriging and disjunctive kriging. However, there are not used in the present study, and hence are not presented herein.

The validity of all the assumptions used in kriging is checked using cross validation tests. These tests are also useful to choose the best model from the candidate models. A method known as cross-validation (also termed as jackknifing) is used. In this method, kriging is performed at all the data points, ignoring e h one of them in turn one by one. Statistical analysis of the kriging errors and the standardized errors, also known as reduced errors, is carried out to verify that there is no systematic over or under estimation and the errors are consistent with the corresponding standard deviations. The following four tests are generally performed.

a) For the estimates to be unbiased. The average kriging error (or mean error, ME) must be close to zero

$$\mathrm{ME} = \frac{1}{N}\sum_{i=1}^{N}[z^*(x_i) - z(x_i)] \approx 0 \tag{18}$$

where,

ME = the average kriging error

$z^*(x_i)$ = the estimated value of x_i

$z(x_i)$ = observed value at x_i

N = Number of observation points

b) Mean square error must be minimum. This error provides an overall effective measure of the accuracy of the model.

$$\frac{1}{N}\sum_{i=1}^{N}\left[z^*(x_i) - z(x_i)\right]^2 = Minimum \tag{19}$$

c) The mean of reduced errors (kriged reduced mean error, KRME) must be close to zero.

$$\mathrm{KRME} = \frac{1}{N}\sum_{i=1}^{N}\left\{\left[z^*(x_i) - z(x_i)\right]/\sigma_{ki}\right\} \approx 0 \tag{20}$$

where,

σ_{ki} = kriging standard deviation at point x_i

d) Variance of the reduced errors (kriged reduced mean square error, KRMSE) must be close to one

$$\text{KRMSE} = \frac{1}{N}\sum_{i=1}^{N}\left\{\left[z^{*}(x_i) - z(x_i)\right]^2/\sigma_{ki}^2\right\} \approx 1 \tag{21}$$

where,

σ^2_{ki} = kriging variance at point x_i

The last two tests verify the theoretical consistency of the selected semivariogram model.

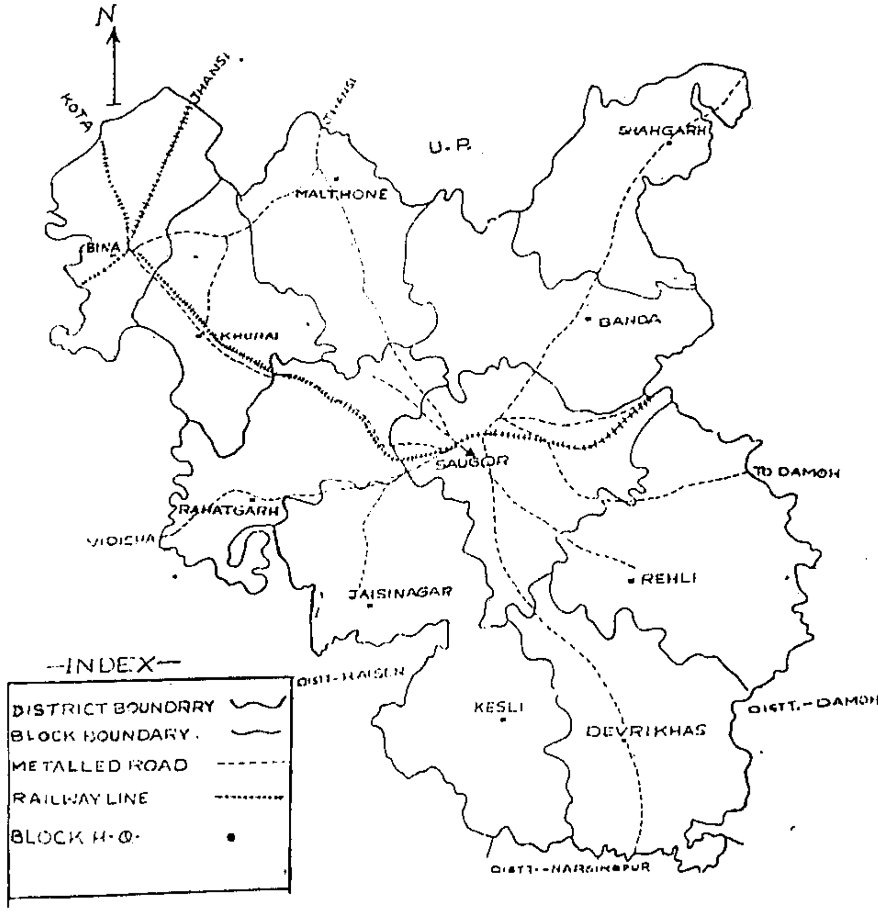
3.0 STUDY AREA

Sagar district (Fig.4), having a geographical area of 10.23 lakhs hac, falls in the state of Madhya Pradesh between 78°00′ to 79°16′ East longitude and 23°10′ to 24°19′ North latitude covering Survey of India toposheet No. 54L, 54P, 55I, and 55M. Out of the 10.23 lakh hac land, the district has a culturable land of about 7.29 lakh hac.

The area enjoys a pleasant climate in the subtropical climate zone. Moderate to extreme heat is observed during the summer season in the area. The mean maximum temperature varies from 40°C to 42°C, while mean minimum temperature varies between 11°C to 8°C. The area is influenced by south-west monsoon. The rain starts from mid of June and goes upto mid of October. The mean annual rainfall for Sagar station is 1215.7 mm.

The area falls under Bundelkhand plateau as per broad physiographical classification. The physiographically the area falls under Piedmont plain pediment, longitudinal ridges and flood plain. The topography of the area is rolling to undulating with plain. The land slope is characterised by flat topped hillocks. This topography is a result of the variation in hardness of different flows. The hard portions forming the top of the terraces and hillocks, the soft part are eroded away.

Due to rolling and undulating topography, the upland area is having excessive surface drainage which has resulted in severe loss of surface soil and hence expose the parent material. The soils removed from uplands get accumulated on the valley land. The valley lands are moderately to poorly drained. The natural drains are limited in number and the drainage density is low. The uplands have a dendetric drainage pattern.



MAP OF DISTRICT SAGAR

Sagar has been mapped under lower and upper Vindhyan system in the map of geological mineral map of Madhya Pradesh. The important rock formation occurring in the area are Vindhyan sand stone, Quartizitic sand stone, lime stone and Deccan trap called basalt. Basalt rocks overlie the Vindhyan sand stone. Lower Vindhyan are represented by quarzitic sand stone and shales where as upper Vindhya consists of sand stone and shales with subordinate limestone. Lameta limestone is also found in lower reaches. This basic rock formation mainly governs the soil characteristics of the watershed area.

The major part of the area is occupied by basaltic rocks belongs to middle Cretaceous to lower Eocene. Inthis basaltic terrain out crops of quartzite sandstone occurs as intens and these belong to the Rewa group of the Vindhyan super group. The other formations which are found in the area are Lametas and Bijawar.

The soils of the area have been derived from basaltic parent material and are classified under medium black soils under broad classification of Indian soils. Alluvium is also found in the area along the streams and river banks. The area falls in predominantly mono-cropped area, Rabi being the principle crop season. The main crops grown in the area Wheat, Gram, Masoor, Jawar, Makka etc.

The area comes under the major east Yamuna Basin and subbasins of river Bewas, Dhasan and Sonar. The river Bewas, Dhasan and Sonar are the main rivers in the area. These rivers are perennial in nature but the discharge decreases considerably during the summer season. There are many small perennial and non-perennial streams which drain the surface water of the area to main rivers during the monsoon. There is one reservoir known as Bila reservoir in the Shahgarh block.

Groundwater in the area occurs generally under water table conditions. The main natural recharge to the groundwater in the area is from precipitation, influent seepage from streams during rainy season and seepage from small tanks, with maximum contribution from rainfall.

Groundwater survey work in the district was started by M.P. state Groundwater surveying Department, unit Sagar, in the year 1973. Initially 50 permanent observation wells were selected in the whole district scattered in different blocks and in different formations of geology. From the 1985, 50 more observation wells were selected. Now the static water level of 100 wells is recorded. The data used in this report has been collected from the above said department.

4.0 RESULTS AND DISCUSSION

The methodology described above has been applied to the groundwater level data of Sagar district of Madhya Pradesh (India).

Fig.5 shows the location of the observation points (for which groundwater level data is available) for premonsoon of 1995, with the number identifying well identity. The total number of observations wells set up are 100, but for this study only 90 wells are used as only on these points water level data was available due to the various reasons; namely, nonavailability of topographical level of some observation points, nonrecording of data at some wells in that year etc. The groundwater elevation at each of the observation point is calculated by subtracting the depth to groundwater from the topographic elevation of corresponding observation point. The basic statistics, pertaining to the data, such as the mean, variance, coefficient of variance (CV) and minimum and maximum value of the observed groundwater levels are shown in Table 1.

Table 1 Basic statistics of data set

Sr. No.	Parameter	units	Quantity
1.	No. of data	---	90
2.	Mean	meter	448.70
3.	Variance	$meter^2$	2169.83
4.	Coeff. of Vari	---	0.10
5.	Minimum value	meter	342.87
6.	Maximum value	meter	579.85

Fig. 5. Location of Observation Points

4.1 Semivariogram analysis

Omnidirectional semivariogram is calculated for premonsoon period for the year 1995. As the observation points are not uniformly distributed in the study area, tolerance is introduced in distance. A lag distance of 7.5km and a distance tolerance of 3.75km are used to calculate the experimental omnidirectional semivariogram and the same is shown in Fig.6.

The shape of the experimental semivariogram shows the presence of nugget effect, sill and range. Structural analysis is carried out on this semivariogram with a lag distance of 75km. The shape of the semivariogram, shown in Fig. 6, indicates that spherical, exponential, and gaussian model can be fitted to it. Weighted least square error method was used to fit these models. The parameters of the fitted models are shown in Table 2. The experimental and fitted semivariogram are shown in Fig. 7.

Table 2. Fitted Parameters of Different Models

Sr. No.	Model	C_o	C	a	Standard Error
1.	Spherical	0.0	3244.5	75.5	181.9
2.	Exponential	0.0	6183.0	87.25	244.1
3.	Gaussian	307.6	2906.8	36.25	125.4

4.2 Cross validation

To check the validity of all the assumptions made in the development of theoretical model and the estimation of model parameters, cross validation is carried out on the data. The different model with above parameters for cross validation. Results of Jackknifing procedure (in which the values are predicted in turn at all the known

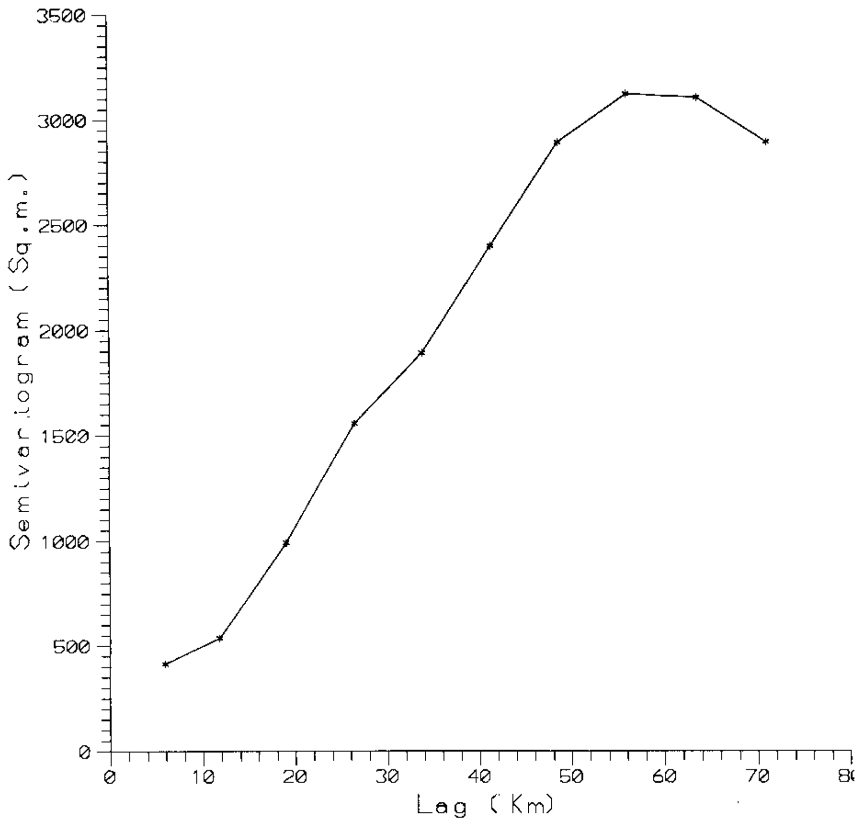


Fig.6 Experimental Semivariogram

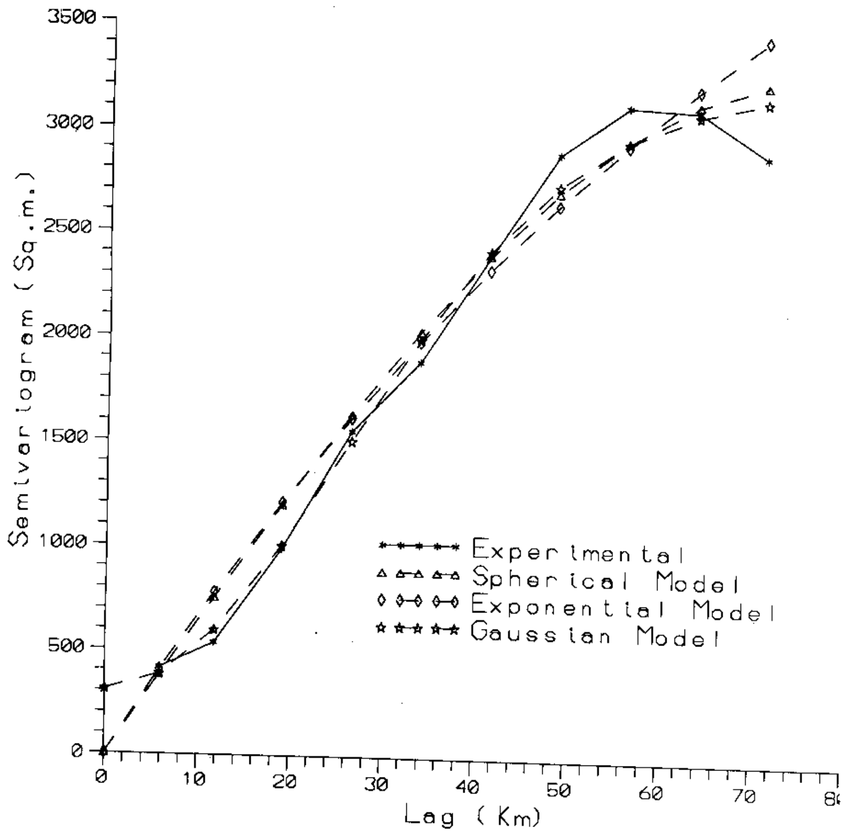


Fig.7 Experimental and fitted Semivariogram

observation points using all the data points except the one point at which prediction is being made) for the data are given in Table. 3.

Table 3 Cross validation result

Sr. No.	Parameter	Model		
		Spherical	Exponential	Gaussian
1.	ME	0.458	0.629	-0.115
2.	KMSE	21.658	21.889	23.01
3.	KRMSE	0.971	0.901	1.279
4.	KRME	0.005	0.01	-0.016

The cross validation results shows that for all fitted models the mean error (ME) is very near to zero, root mean square error (RMSE) is very low as compared to the standard deviation of the data i.e. 46.58 m, kriged reduced mean square error (KRMSE) is near to 1, and a kriged reduced mean error (KRME) is near to zero. The above cross validation results show that the chosen models and their parameters are adequate.

The cross validation results also shows that for the spherical model, The KRMSE is very near to one. All other three parameters by spherical model are also comparable to parameters of other models. So, the spherical model was chosen as the final model to be use in kriging.

4.3 Kriging

For interpolated estimation of groundwater level at any unsampled location, kriging can be applied. The study area is divided into a square grid of 25 km and groundwater levels are estimated at each of the grid nodes using the finally selected spherical model. These estimated level values are used with the SURFER software to draw the contour maps. The contour map of the groundwater levels so obtained are shown in Fig. 8.

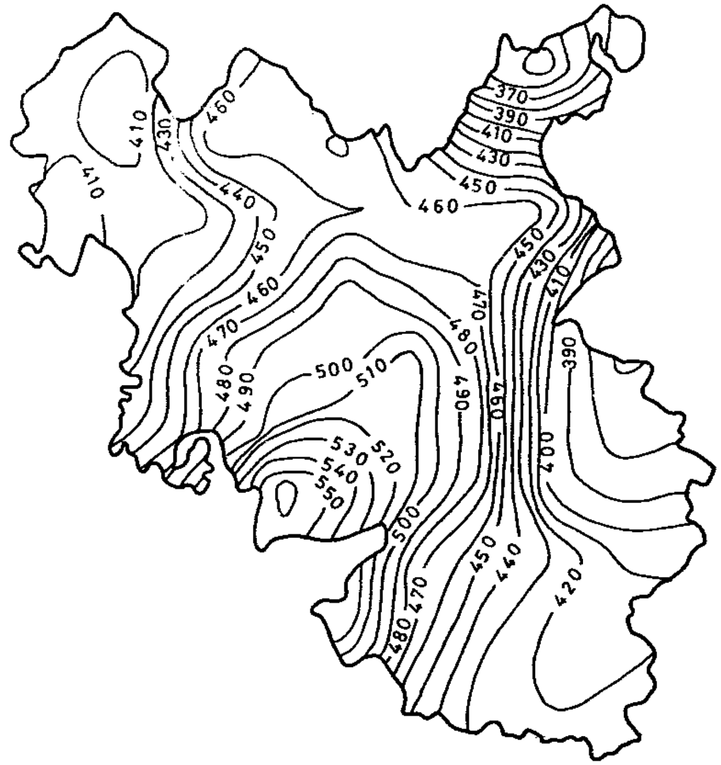


FIG. 8 KRIGED GROUNDWATER LEVEL CONTOUR MAP.

5.0 CONCLUSIONS

Interpolation of ground water levels is of significant importance in agricultural and hydrologic contexts. A variety of interpolation techniques are in use. In this study, kriging, a type of geostatistical techniques, is applied to the groundwater level data of premonsoon period of 1995 in the Sagar district of Madhya Pradesh. The spherical model is found to the best model representing the spatial variability of groundwater level data. The groundwater levels are found to be related upto a distance of 75km.

6.0 REFERENCES

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